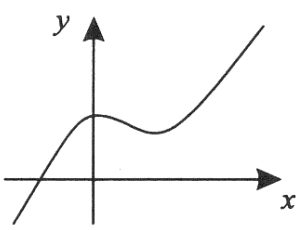
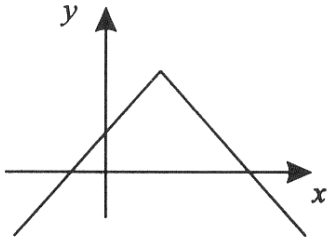
FEATURES OF GRAPHS

Continuity of Functions

A function is **continuous** if its graph can be drawn without lifting the pen off the page, i.e. there are **no breaks** in it.

The graphs below are of continuous functions:





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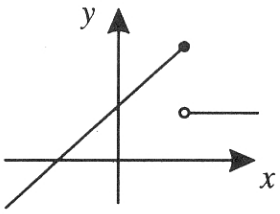
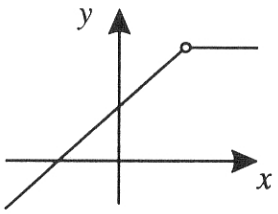
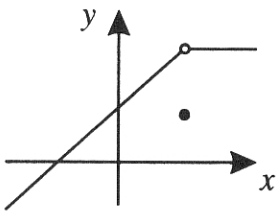
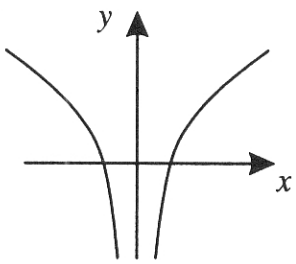
The graphs below are of **discontinuous** functions:







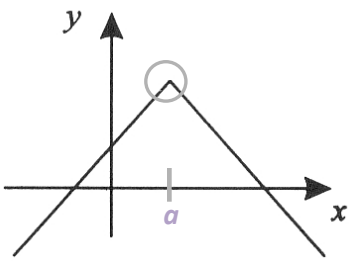


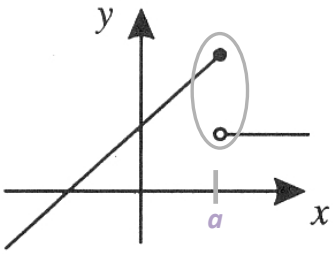
Differentiability of Functions

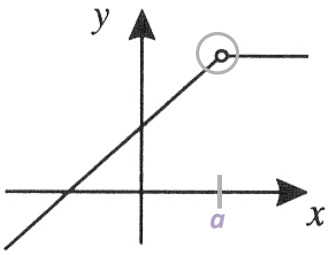
A function cannot be differentiated at a point if:

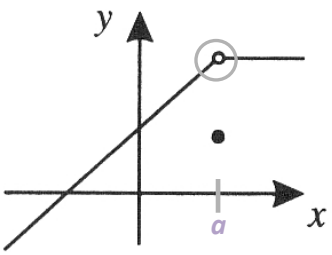
- It is not continuous at that point
- There is a sharp 'corner' or 'point' in the curve at that point
- The function does not exist at that point i.e. there is a 'hole' or a 'break'.

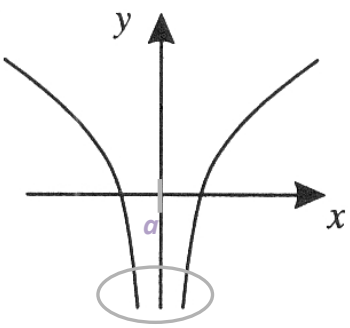
The functions below are all not differentiable at $x = a$ due to breaks or sharp corners in the curve, which have been circled.











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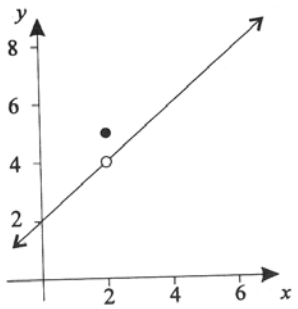
FEATURES OF GRAPHS

Graphs and their limits

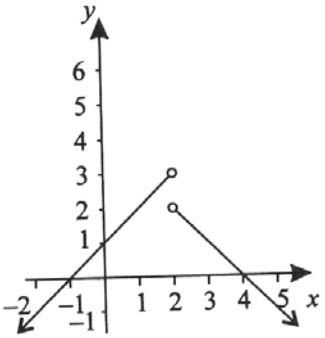
The limit of a graph at point a is the **y-value** the graph **approaches** from either side of point a.

1) The limit for the graph on the right at point x = 2 is 4.
This can be written as $\lim_{x\to 2} f(x) = 4$.



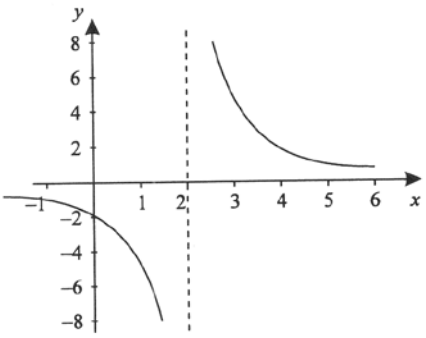
A limit exists at x = 2, even though there is a 'hole' exactly at x = 2. The function need only **approach** the same y-value from either side of point a to have a limit a point a, and the limit is the y-value the function **approaches**, not the actual value f(2) = 5.

2) The limit for the graph on the right at point x = 2 does not exist.



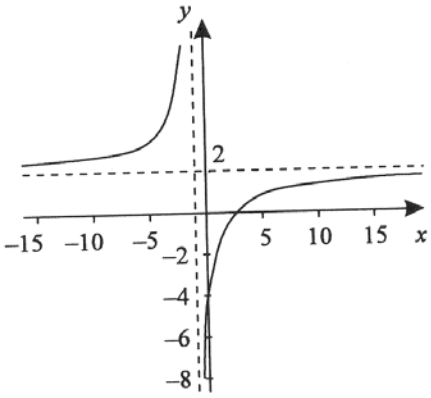
There is a vertical gap at x = 2. The function approaches the y-value of 3 to the left of x = 2, but approaches the y-value of 2 to the right of x = 2. Therefore, no limit exists since the function does not approach the same y-value on either side of x = 2.

3) The limit for the graph on the right at point x = 2 does not exist.



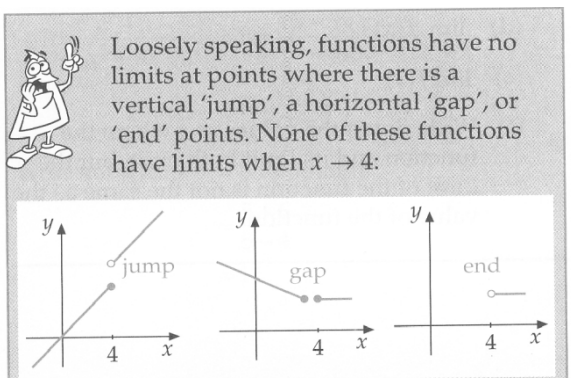
The graph has a vertical asymptote at x = 2. The function approaches $-\infty$ to the left of x = 2, but approaches $+\infty$ to the right of x = 2.

4) The limit for the graph at point x = -1 does not exist.



The limit for the graph as $x \to \infty$ is 2.
$\lim_{x\to\infty} f(x) = 2$

The limit for the graph as $x \to -\infty$ is 2.
$\lim_{x\to-\infty} f(x) = 2$



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